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Channel Capacity in a Dynamic Random Waypoint Mobility Model

Faeik T. Al Rabee, *Student Member, IEEE*
Department of Electrical Engineering
University of South Florida
Tampa, Florida 33620, USA
faeiktayseer@mail.usf.edu

Ashraf Al-Rimawi, *Member, IEEE*
Department of Electrical and Computer Engineering
Birzeit University, BZU
Palestine
aalrimawi@birzeit.edu

Richard D. Gitlin, *Life Fellow, IEEE*
Department of Electrical Engineering
University of South Florida
Tampa, Florida 33620, USA
richgitlin@usf.edu

***Abstract*— In this paper the channel capacity for a dynamic random waypoint (RWP) mobility model of a Rayleigh fading channel is derived. A maximum ratio combining (MRC) diversity receiver and the effect of the number of branches, *N*, on the channel capacity is determined. As expected, by increasing the number of diversity branches, the resulting channel capacity is increased until the capacity is saturated. For example, the channel capacity for $N = 3$ is increased by 38.3% compared to no diversity ($N = 1$) for the same value of the average received signal-to-noise ratio ($\overline{\text{SNR}}$) but increasing *N* beyond 12 provides minimal gain. The channel capacity is compared with the classic Shannon capacity of the AWGN channel and with the well-known static model Rayleigh fading channel capacity. The channel capacity of the RWP Rayleigh channel is reduced by 10% compared to the AWGN Shannon capacity for a $\overline{\text{SNR}}$ of 20 dB. As expected, the AWGN channel capacity has a larger channel capacity as it is not affected as severely by fading as in the RWP mobility model. By contrast, the RWP model shows a slight improvement in channel capacity in comparison with the static model Rayleigh fading channel, since it will not be affected by severe fading for as long a time period as the static Rayleigh model. For example, the proposed model channel capacity increases to 6.11 bps/Hz whereas it is 5.87 bps/Hz for the static model Rayleigh fading channel capacity at the same $\overline{\text{SNR}} = 20$ dB with increasing of 4%.**

Keywords— Channel capacity; mobility models; random waypoint (RWP) model; Rayleigh fading channel; maximum ratio combining (MRC).

I. Introduction

In the coming years, the fifth generation (5G) is expected to be the most widely used wireless communications system which will provide low power consumption, high spectral efficiency, and low latency. In order to meet the requirements of future wireless communications systems and networks, a new standard of 5G system is being created [1]. In addition, as 5G systems are expected to support existing applications, it will be able to integrate new solutions in order to meet the dramatic increase for high data rate [2].

As a result, the performance behavior of the 5G wireless communications system is a key parameter in improving the system networks behavior [3]. The performance of 5G can be characterized using several parameters as capacity, outage probability, and spectral efficiency, etc [4]. The studying of these performance parameters gives an indication about enhancing the wireless networks quality and capacity. Channel capacity represents a basic measure of performance in information theory as it defines the upper bound of the maximum transmission rate of data at a vanishingly small bit error rate (BER). In 1948, Shannon derived the channel capacity in a given AWGN environment [5] and the channel capacity for fading environments was derived several decades later [6]-[9].

Moreover, channel capacity is used to study the effect of multipath fading statistical models where the received signal power is characterized [3]. As discussed in the literature, the static wireless networks have mostly been considered by these models. In this case, the average received power is given as a function of the transmitted power, transmitter-receiver distance, and path loss exponent. In static models, the average received power is constant since the transmitter-receiver distance is constant. Depending on that, the power follows an exponential distribution and the received signal is Rayleigh distributed because of the multipath fading that varies the received signal at the receiver [10].

Antenna diversity has been proposed as a technique to mitigate the multipath fading effect by improving the signal-to-noise ratio (SNR) at the receiver side and increasing the wireless system capacity [11]. Various diversity techniques have been discussed in the literature [12]. Maximum ratio combining (MRC) technique is used in this paper as one of the diversity techniques [13].

On the other hand, in mobile communication systems, the distance between the transmitter and receiver is not constant and it follows a somewhat random behavior. Furthermore, the received signal varies with time due to nodes mobility, distance dependent path loss, and multipath fading which results in time varying received power [12]. In this case, the received power does not follow an exponential distribution as followed in the wireless static networks [10]. Several mobility models have been discussed in the literature, for example random waypoint (RWP) and random walk (RWM) models [14]-[16]. These models are designed to describe how the mobile users (nodes) change their velocity and location over time in addition to their movement pattern [15]. In this paper, the RWP model is considered as the dynamic model for deriving the channel capacity as discussed in the next section.

So far, a lot of research efforts in the literature have been devoted to the study of wireless communication networks capacity in terms of static model only [17]-[19]. In [17], the authors have been presented an architecture for analyzing the network capacity when Device-to-Device (D2D) communications share the channel resources with cellular links. The authors in [18] have been studied the achievable transmission capacity of secondary users in heterogeneous networks (HetNets). In [19], the optimal D2D transmission capacity and density in different bands have been studied.

To our knowledge, it is notable that these studies have not been considered the dynamic case to find the channel capacity of the wireless system network. Recent research [3] and [20], have studied the outage probability in a dynamic model. In [3], the distribution probability density (PDF) function and the outage probability of dynamic mobility model has been derived over a (η-μ) fading channel [21]. In [20], an expression of the outage probability in dynamic mobility model over a Rayleigh fading channel has been derived. This research discussed the outage probability not the channel capacity. The main contribution of this paper can be summarized as follows:

- Deriving an expression for the channel capacity of a wireless network in RWP dynamic mobility model over a Rayleigh fading channel using a MRC reception diversity technique. The capacity is given in terms of the well-known incomplete gamma function [22] which is easily implemented in computing software.
- Evaluating the effect of the number of branches in a MRC on the resulting channel capacity.
- Comparing the derived result of the channel capacity with the classic Shannon capacity of the AWGN channel and with the well-known static model Rayleigh fading channel capacity.

The remainder of this paper is organized as follows. Section II describes the channel model illustrating the concept of using a RWP model with a MRC to derive the channel capacity. The numerical results, which are related to the derived capacity expression in Section II, are presented in Section III. Finally, Section IV summarizes the conclusions of the paper.

II. CHANNEL MODEL

In this section, a channel model is introduced to derive the channel capacity of the dynamic RWP mobility model. As discussed in Section 1, in a mobile wireless network, the nodes are mobile and because of that the distance between these nodes is random. As a result, multipath fading is time varying in nature and the received power does not follow an exponential distribution as in static network. So, it is appropriate to investigate the mobility effect and evaluate the performance of mobile wireless network in terms of the channel capacity.

In order to conduct this study, one of the widely available dynamic mobility models is assumed, which is the RWP model [15]. In the RWP model, each mobile user (node) selects one location in the network as a destination. Then, this node travels with random and uniform velocity toward its destination at velocity [0, V_{max}], where V_{max} is the maximum velocity for each mobile node [20]. The direction and the velocity of each mobile node are independently chosen. After reaching its destination, the mobile node stops for a random period of time. After this time period, the mobility node selects a new random destination and start moving towards it in the same scenario in the previous case. More details about RWP model are available in [14].

Since the channel in this study is assumed as a fading channel, one of the major factors that affects the modeling of the fading channel is the distributions that are used to characterize the channel. In our model, the channel is assumed to have a Rayleigh distribution. With an optimum combiner diversity technique, MRC, with *N*-branches, results in the SNR being the sum of the SNR of each individual diversity branch [13]. Therefore, *N*-branches of MRC are assumed at the receiving node to mitigate the effect of fading channel and improve the channel capacity of our proposed model. This improvement is investigated in Section III containing the numerical results.

In summary, an expression for the channel capacity of a dynamic RWP mobility model of a Rayleigh fading channel using MRC at the receiver will be derived. For deriving this result, the formula for channel capacity *C* of a fading channel in [23] is used. As mentioned above where the received power depends on the transmitter-receiver distance, this means that the distance from the transmitter to the receiver is reflected in the received power which can translate to the received SNR [3]. Therefore, the channel capacity *C* of a fading channel is given as:

$$C = B \int_{\Omega_0}^{\infty} log_2\left(\frac{r}{\Omega_0}\right) P_r(r)\, dr, \tag{1}$$

where *B* is the channel bandwidth in [Hz]. The parameter *r* is the random variable that defines the distance distribution between the mobile device (node) and the access point. The parameter $P_r(r)$ is the probability density function (PDF) of received power. And Ω_0 represents the average received signal power.

To evaluate (1) it is necessary to determine $P_r(r)$ based on the assumed model As a result, the received power PDF of a RWP mobility model with an *N*-branch MRC is given in [20] as

$$P_r(r) = \frac{6\, P_t^{\frac{2}{\alpha}}\, r^{-\left(1+\frac{2}{\alpha}\right)}}{\alpha\, \Gamma(N)} \left[\gamma\left(\left(N+\frac{2}{\alpha}\right), \frac{r}{P_t}\right) - \left(\frac{r}{P_t}\right)^{-\left(\frac{1}{\alpha}\right)} \gamma\left(N+\frac{3}{\alpha}, \frac{r}{P_t}\right)\right], \tag{2}$$

where P_t is the transmitted power. The parameter α is the path loss exponent. The $\Gamma(N)$ is the Gamma function which is defined in [22]. Furthermore, the function $\gamma\,(:,:)$ is called the lower incomplete gamma function [24].

By substituting the PDF of the RWP dynamic model of (2) in (1), then the resulting channel capacity of the RWP channel is given as

$$C = \int_{\Omega_0}^{\infty} log_2\left(\frac{r}{\Omega_0}\right) \frac{6\, P_t^{\,2/\alpha}\, r^{-(1+2/\alpha)}}{\alpha\,\Gamma(N)} \left[\gamma\left(\left(N+\frac{2}{\alpha}\right), \frac{r}{P_t}\right) - \left(\frac{r}{P_t}\right)^{-\left(\frac{1}{\alpha}\right)} \gamma\left(N+\frac{3}{\alpha}, \frac{r}{P_t}\right)\right] dr. \qquad (3)$$

Using [25] and Mathematica results to solve the integral in (3), the dynamic RWP mobility model channel capacity of Rayleigh fading using N-branches of MRC is given as:

$$C = \frac{1}{6\,\Gamma(N)} B\, \Omega_0^{-3/\alpha} P_t^{2/\alpha} \Bigg[9\, \Omega_0^{1/\alpha} \alpha \left(\Gamma\left(\mathrm{N}+\frac{2}{\alpha}\right) - \gamma\left(\left(N+\frac{2}{\alpha}\right), \frac{\Omega_0}{P_t}\right)\right) - 4\,\alpha P_t^{\frac{1}{\alpha}} \left(\Gamma\left(\mathrm{N}+\frac{3}{\alpha}\right) - \gamma\left(\left(N+\frac{3}{\alpha}\right), \frac{\Omega_0}{P_t}\right)\right) + \Omega_0^{\frac{3}{\alpha}} \left(\frac{1}{P_t}\right)^{\frac{2}{\alpha}} \Bigg(-\left(-9+4\left(\frac{1}{P_t}\right)^{\frac{1}{\alpha}} P_t^{\frac{1}{\alpha}}\right) \alpha\, \gamma\left(N, \frac{\Omega_0}{P_t}\right) - 6\left(-3+2\left(\frac{1}{P_t}\right)^{\frac{1}{\alpha}} P_t^{\frac{1}{\alpha}}\right) meijerG\left[\{\{\},\{1,1\}\},\{\{0,0,N\},\{\}\}, \frac{\Omega_0}{P_t}\right]\Bigg)\Bigg], \qquad (4)$$

where the 'meijerG' function [26] is a very general function which reduces to simpler special functions in many common cases.

III. Numerical Results

Based on the result in (4), Mathematica software [27] has been used to investigate the relation of the channel capacity versus $\overline{\text{SNR}}$, where the thermal noise fixed at unity. The path loss exponent α has been assumed to be 3. The results have been analyzed by normalizing the resulting channel capacity with respect to the channel bandwidth B. In Fig. 1, the effect of the number of MRC branches, N, on the resulting proposed model channel capacity has been depicted.

As expected in Fig.1, increasing the number of branches, N, leads to improved channel capacity. Additionally, it can be observed that when N is less than two ($N \leq 2$), the dominant channel capacity effect is $\overline{\text{SNR}}$. With these results we find, for example, in the absence of diversity N =1, the proposed channel capacity is equal to approximately 6 bps/Hz when $\overline{\text{SNR}}$ = 20 dB. As N is increased, for example N = 3, the channel capacity is increased to 8.3 bps/Hz for the same value of $\overline{\text{SNR}}$, respectively. Therefore, the channel capacity of a dynamic RWP Rayleigh channel for N = 3 is increased by 38.3% compared to no diversity. As a result, by combining more diversity branches, the channel capacity is increased since the effect of the fading will be decreased. Furthermore, as expected for a large N, the C_{RWP} saturates since the maximum effective receiver diversity is realized.

A comparison between the RWP channel capacity (C_{RWP}) with the AWGN channel Shannon capacity (C_{AWGN}) and the static model Rayleigh fading channel capacity ($C_{Rayleigh}$) is shown in Fig. 2. The figure shows that the AWGN channel capacity is better than the RWP model channel capacity. For example, for $\overline{\text{SNR}}$ = 15 dB, C_{AWGN} = 5.1 bps/Hz, whereas C_{RWP} = 4.48 bps/Hz. For $\overline{\text{SNR}}$ = 20 dB, C_{AWGN} = 6.7 bps/Hz, whereas C_{RWP} = 6.11 bps/Hz. Therefore, the channel capacity of the suggested dynamic RWP model is reduced by 14% for $\overline{\text{SNR}}$ = 15 dB and by 10% for $\overline{\text{SNR}}$ = 20 dB, respectively. The AWGN channel capacity has a larger channel capacity as it is not affected as severely by fading as in the RWP mobility model.

However, the comparison of the resulting channel capacity (C_{RWP}) with the static model Rayleigh fading channel capacity ($C_{Rayleigh}$) shows that the C_{RWP} is slightly larger than the $C_{Rayleigh}$ for the same $\overline{\text{SNR}}$. For example, for $\overline{\text{SNR}}$ = 15 dB, $C_{Rayleigh}$ = 4.31 bps/Hz, whereas C_{RWP} = 4.48 bps/Hz. For $\overline{\text{SNR}}$ = 20 dB, $C_{Rayleigh}$ = 5.87 bps/Hz, whereas C_{RWP} = 6.11 bps/Hz. Therefore, the channel capacity of the suggested dynamic RWP model (C_{RWP}) is approximately increased by 4% for $\overline{\text{SNR}}$ = 15 dB and $\overline{\text{SNR}}$ = 20 dB, respectively.There is a negligible improvement in the RWP channel capacity compared to the static model of Rayleigh fading channel capacity. The reason is that severe fading will not affect the RWP model for as long a time period as it affects the static Rayleigh model.

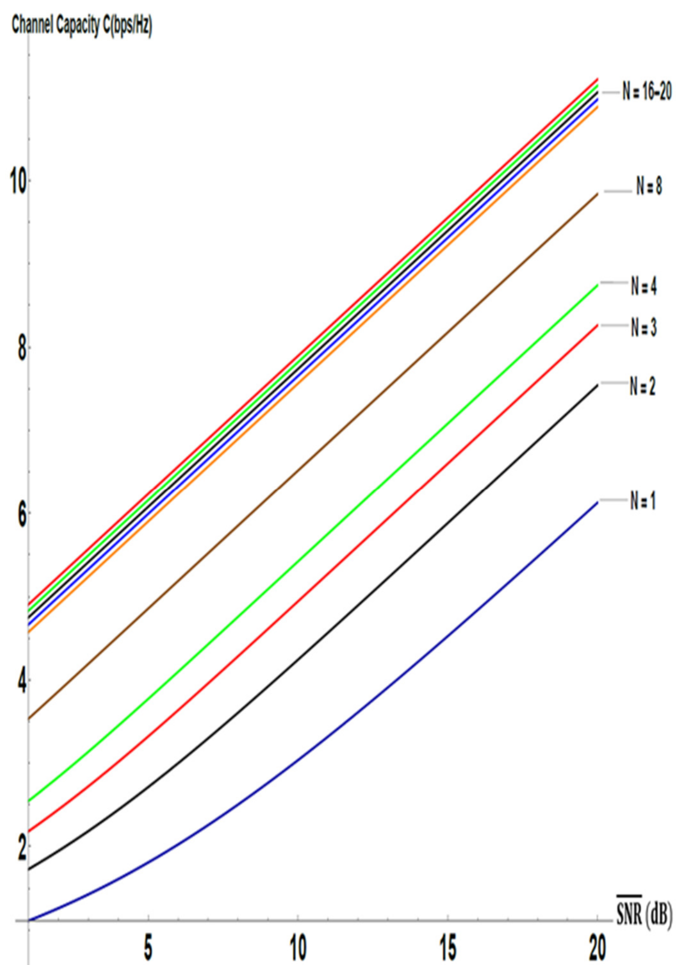

Fig. 1. The effect of the number of branches, N, on the RWP mobility model channel capacity.

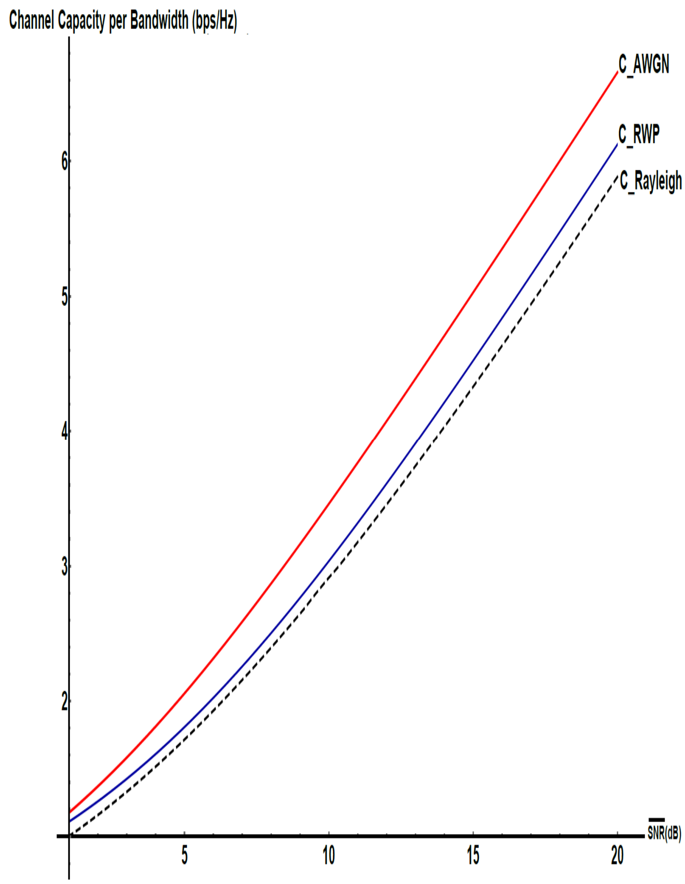

Fig. 2. The RWP mobility model channel capacity in comparison with the AWGN and the static model Rayleigh fading channel capacities.

IV. Conclusion

In this paper, an expression for the channel capacity of a dynamic RWP mobility model based on a Rayleigh fading channel, with a MRC receiver, has been derived. The result shows that the channel capacity increases as the number of MRC branches, *N*, is increased, e.g., the resulting channel capacity is increased from 6 bps/Hz at $N = 1$ to 8.3 bps/Hz at $N = 3$ for the same value of $\overline{\text{SNR}} = 20$ dB. However, the resulting channel capacity for a large *N* saturates since the maximum effective receiver diversity is realized. In addition, the derived channel capacity result has been compared with the classic AWGN Shannon capacity and with the static model Rayleigh fading channel capacity. The numerical results show that the dynamic RWP mobility channel capacity (C_{RWP}) is lower than the AWGN channel capacity (C_{AWGN}) as expected. For example, the proposed model channel capacity (C_{RWP}) is decreased by 10% compared to the AWGN channel capacity (C_{AWGN}) at the same $\overline{\text{SNR}} = 20$ dB as a result of the AWGN channel capacity is not affected as severely by fading as in the RWP mobility model. However, the comparison of the resulting channel capacity with the static model Rayleigh fading channel capacity $\left(C_{Rayleigh}\right)$ reflects that the derived result shows a slightly better channel capacity for the same $\overline{\text{SNR}}$. For example, for $\overline{\text{SNR}} = 20$ dB, the channel capacity of the RWP model is increased by 4% compared to the well-known result of the static model Rayleigh fading channel capacity. The result is expected since severe fading will not affect the RWP model for as long a time period as it affects the static Rayleigh model.

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